

Harlow Green Community Primary School

Maths Calculation Policy



Concrete apparatus



dienes



place value counters



rekenrek

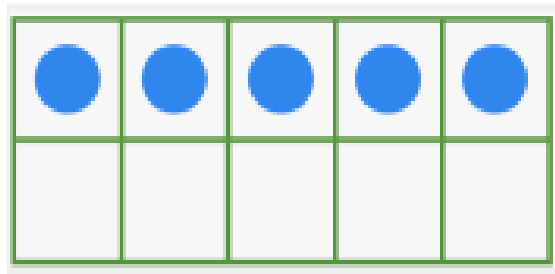


numicon

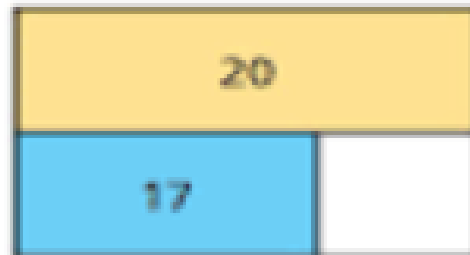
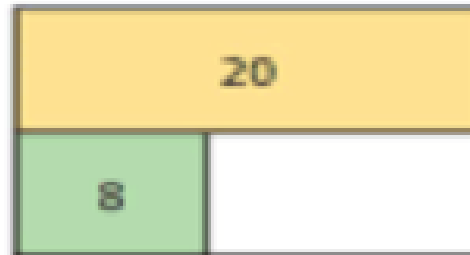


double sided counters

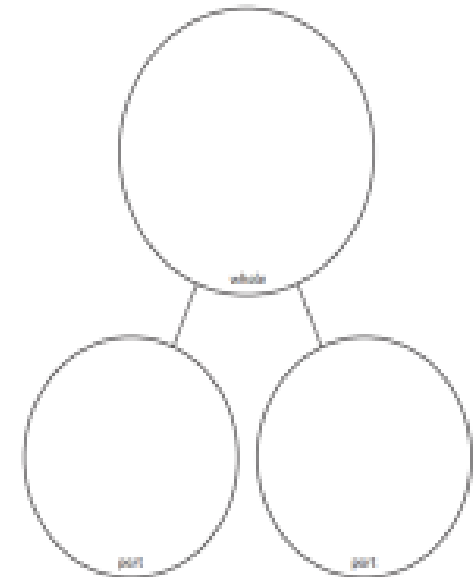
Representations



five / tens frames



bar models



part whole models

Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

hundred square


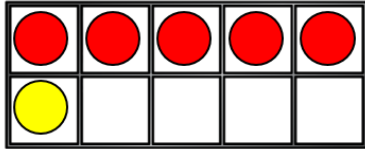
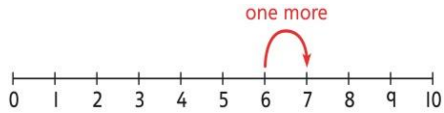
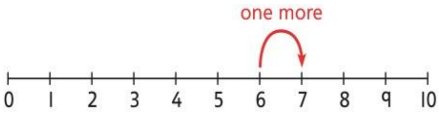
x12	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12
x1	1	2	3	4	5	6	7	8	9	10	11	12
x2	2	4	6	8	10	12	14	16	18	20	22	24
x3	3	6	9	12	15	18	21	24	27	30	33	36
x4	4	8	12	16	20	24	28	32	36	40	44	48
x5	5	10	15	20	25	30	35	40	45	50	55	60
x6	6	12	18	24	30	36	42	48	54	60	66	72
x7	7	14	21	28	35	42	49	56	63	70	77	84
x8	8	16	24	32	40	48	56	64	72	80	88	96
x9	9	18	27	36	45	54	63	72	81	90	99	108
x10	10	20	30	40	50	60	70	80	90	100	110	120
x11	11	22	33	44	55	66	77	88	99	110	121	132
x12	12	24	36	48	60	72	84	96	108	120	132	144

multiplication square

gattegno chart

10,000,000	20,000,000	30,000,000	40,000,000	50,000,000	60,000,000	70,000,000	80,000,000	90,000,000
1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	70	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Year 1 Addition

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Counting and adding one more</p>	<p>Children add another object to a group to find one more</p>  <p>Add one more to a set of multilink cubes.</p> 	<p>Children can use a number line to understand how to link counting on with finding one more.</p>  <p>Children could draw a picture to show one more.</p>	<p>Children can use a number line to understand how to link counting on with finding one more.</p>  <p>Verbally "One more than 4 is 5"</p> $5 + 1 = 6$	<p>Addition Add More And Make Sum Total Altogether Double Near double Half Halve One more, two more...ten more How many more to make...? How many more is ... than ...? How much more is ...?</p>

Understand teen numbers as one ten and "some more"

Recognise a group of 10 objects and count more

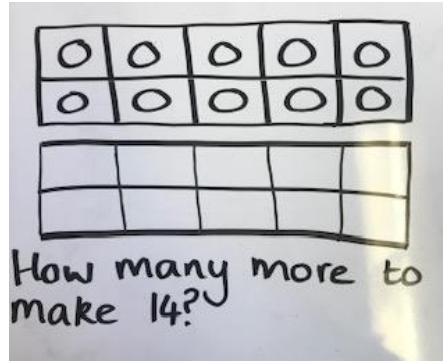


"There are 10 black cubes and 4 grey cubes. 10 and 4 more is 14"

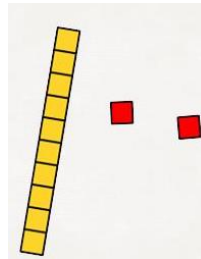
Use a ten frame to support understanding of a complete 10 for teen numbers



"14 is 10 and 4 more"



"14 is 10 and 4 more"

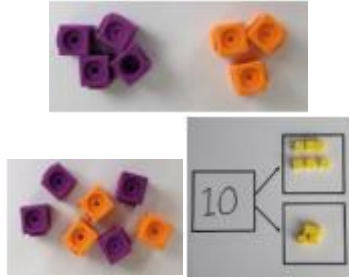


"12 is 10 and 2 more"

"1 ten and 4 ones equals 14"
"10 + 4 = 14"

Combining two parts to make a whole: part-whole model

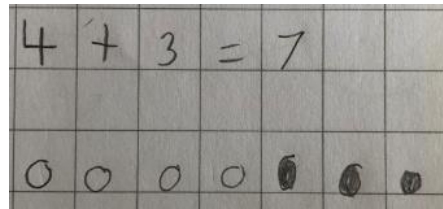
Use part whole model.



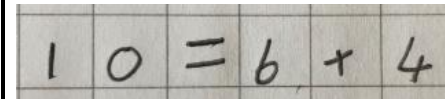
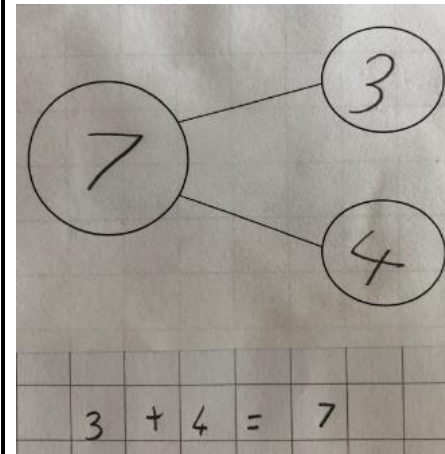
Use cubes to add two numbers together as a group or in a bar.



Use pictures to add two numbers together as a group or in a bar.



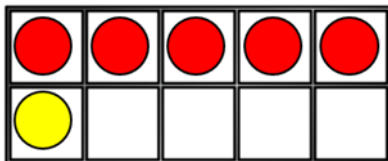
Use the part-part whole diagram to move into the abstract.



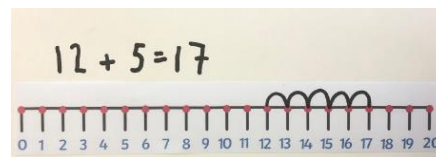
Starting at the bigger number and counting on

Children use knowledge of counting to 20 to find a total by counting on using people of objects

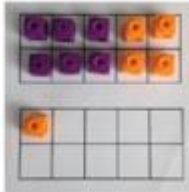
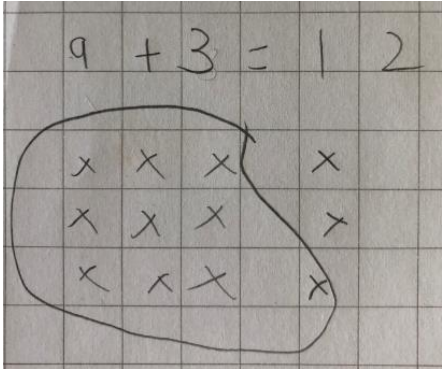
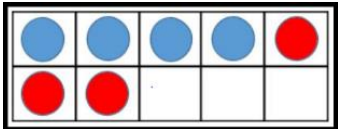

Start with the larger number on the bead string and then count on the smaller number one by one to find the answer.

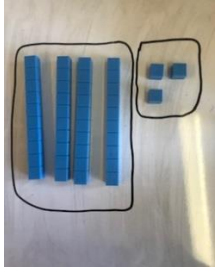

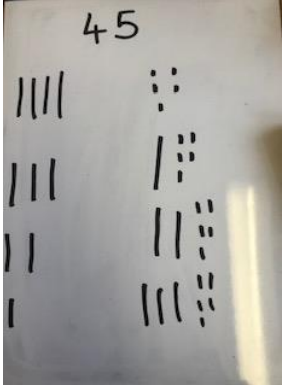
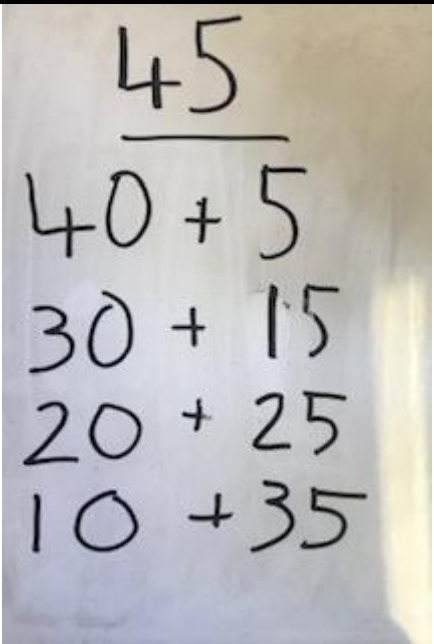
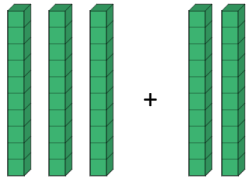
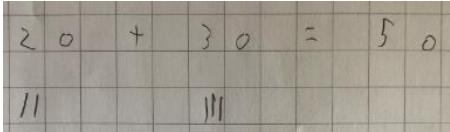
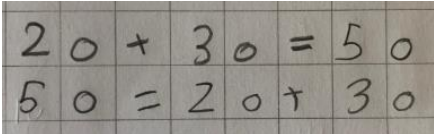


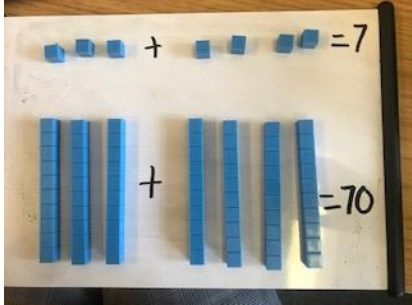
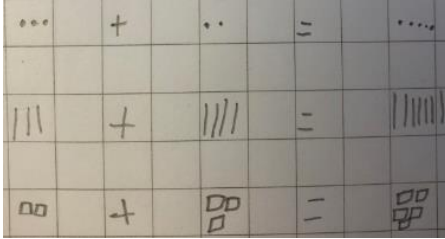

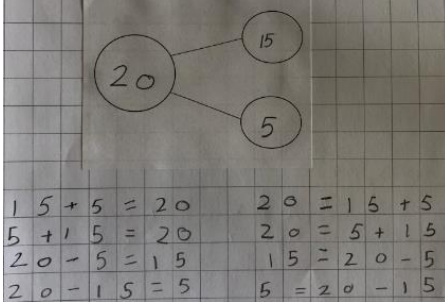
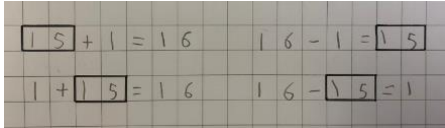

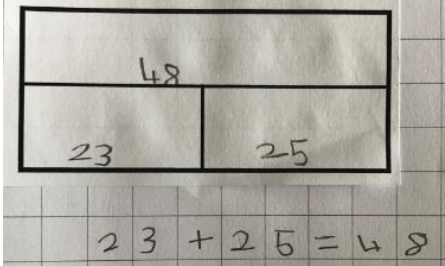
Start with the larger number on the number line and count on in ones or in one jump to find the answer.

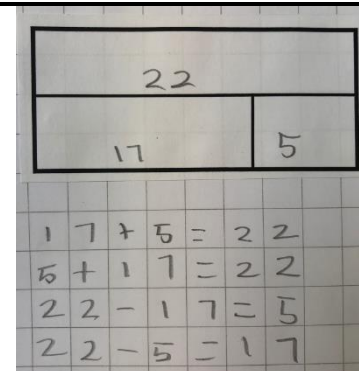
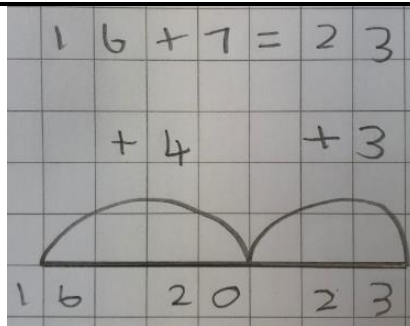
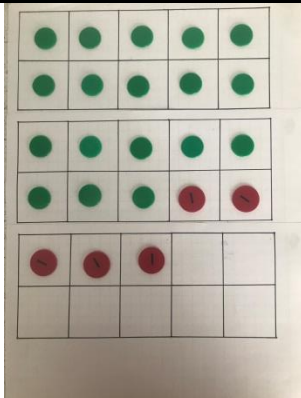


Place the larger number in your head and count on the smaller number to find the answer

<p>Regrouping to make 10/Bridging the 10 using number bonds</p> <p><i>(This is an essential skill for column addition further up the school).</i></p>	<p>Start with the bigger number, partition the smaller number to make 10 and adding on the rest</p> <p>Use tens frames</p> 	<p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10</p> 	<p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10? How many more do I add on now?</p>	
<p>Represent and use number bonds and related subtraction facts within 20</p>	<p>3 more than 4</p> 		<p>Emphasis should be on the language</p> <p>" 1 more than 5 is equal to 6." "2 more than 5 is 7." "8 is 3 more than 5."</p>	
<p>Year 2 Addition</p>				
<p>Objective and strategy</p>	<p>Concrete</p>	<p>Pictorial</p>	<p>Abstract</p>	<p>Vocabulary</p>

<p>Partitioning numbers into 10s and 1s</p>	<p>Dienes to understand unitising of 10s</p> <p>Group objects into 10s and 1s</p> <p>Standard partitioning</p>  <p>$43 = 40 + 3$</p> <p>None standard partitioning</p>  <p>$43 = 30 + 13$</p>	<p>Children may use pictorial representation of dienes to support understanding</p> 		<p>Addition</p> <p>Add</p> <p>More</p> <p>And</p> <p>Make</p> <p>Sum</p> <p>Total</p> <p>Altogether</p> <p>Double</p> <p>Near double</p> <p>Half</p> <p>Halve</p> <p>One more, two more...ten more, one hundred more</p> <p>How many more to make...?</p> <p>How many more is ... than ...?</p> <p>How much more is ...?</p> <p>Equals</p> <p>Is the same as</p> <p>Number bonds/pairs/facts</p> <p>Tens boundary</p>
<p>Adding multiples of ten</p>	<p>$50 = 30 + 20$</p> 			
<p>Using known bonds and</p>		<p>Children draw own representations of H, T and O</p>	<p>"3 ones add 4 ones equals 7 ones therefore 3 <u>tens</u> add 4 <u>tens</u> equals 7 <u>tens</u>"</p>	

<p>unitising to add 10s</p>			<p>$3 + 4 = 7$</p> <p>therefore</p> <p>$30 + 40 = 70$</p>	
<p>Use known number facts</p> <p>Part-part whole model</p>	<p>Children explore ways of making numbers to and within 20.</p> 	 <p>Also with equals sign at the beginning.</p>		
<p>Bar modelling can be used to support</p>	 <p>$10 = 7 + 3$</p> <p>$100 = 70 + 30$</p>	 <p>$23 + 25 = 48$</p>		
<p>Adding a two digit number and ones</p>	<p>Use tens frame to make 10 and then explore the pattern.</p>	<p>Use number line to model</p>	<p>Children use knowledge of composition of number</p>	



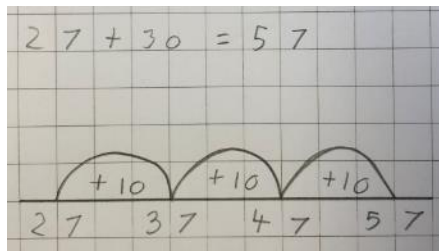
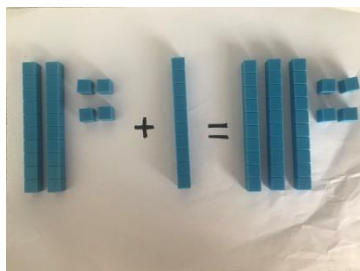
The National Curriculum states *Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers*

There is **NOT** an expectation that children in KS1 will complete formal written methods for addition. However, place value columns can be used alongside pictorial representations to demonstrate place value.

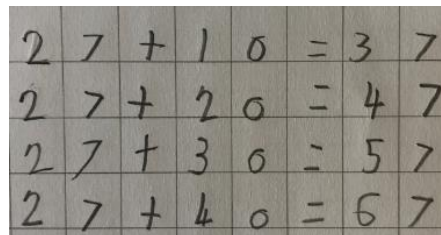
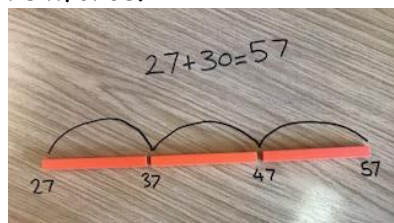


Adding a multiple of 10 to a 2-digit number

Explore that the ones digit won't change.

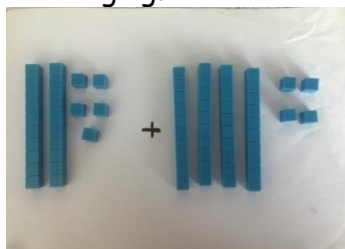


If children struggle to draw "jumps" of similar size, Cuisenaire rods could be used to reinforce.



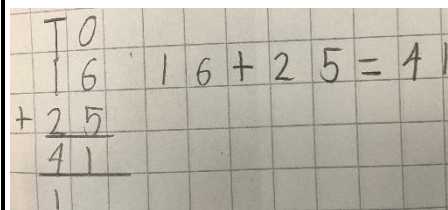
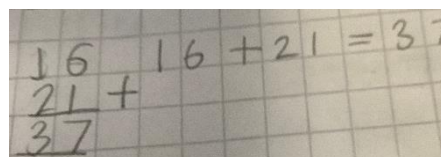
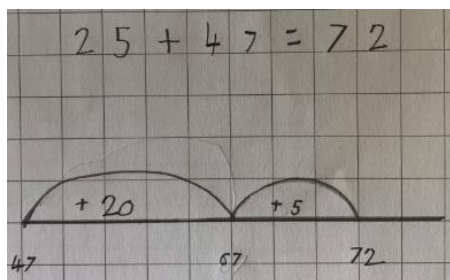
Add two 2-digit numbers

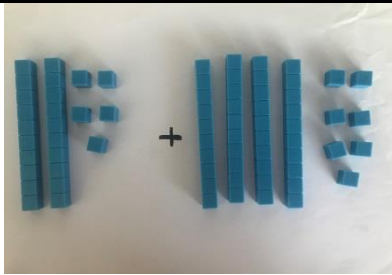
Model using dienes with no exchanging.



Then with exchanging.

Use number line and bridging ten methods if necessary.



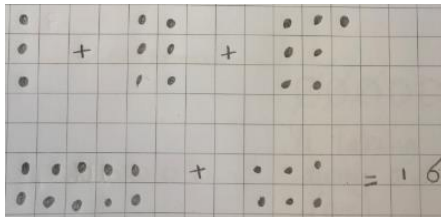


Add three 1 digit numbers

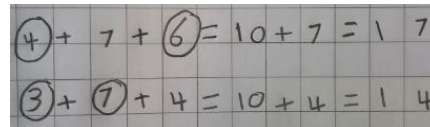
Combine to make 10 first if possible.



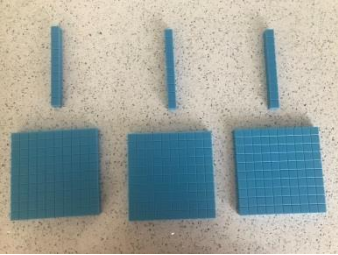
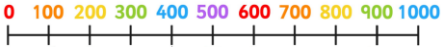
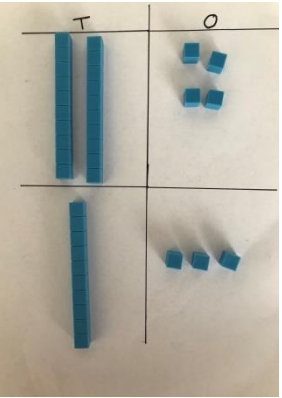
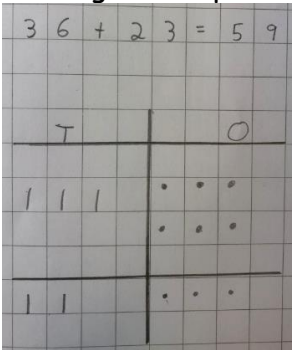
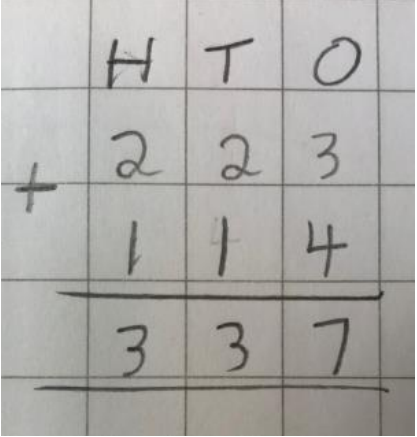
Regroup and draw representations.



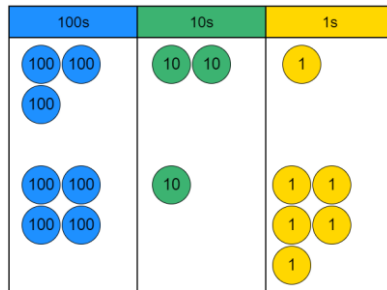
Combine the two numbers that make/bridge the ten then add on the remaining number.



Year 3 Addition

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Understanding place value to 1,000</p>	<p>Understand the cardinality of 100 and the link with ten tens</p>  <p>Unitise 100 and count in steps of 100</p>	<p>Represent steps of 100 on a number line and count up to 1,000 and back</p> 		<p>Addition Add More And Make Sum Total Altogether Double Near double Half Halve One more, two more...ten more, one hundred more How many more to make...? How many more is ... than ...? How much more is ...?</p>
<p>Column addition - no regrouping</p>	<p>Model using dienes or place value counters With 2 digit numbers</p> 	<p>Representing dienes pictorially</p> 	<p>Add the ones first, then the tens and then the hundreds.</p> 	<p>Equals Is the same as Number bonds/pairs/facts Tens boundary/hundreds boundary</p>

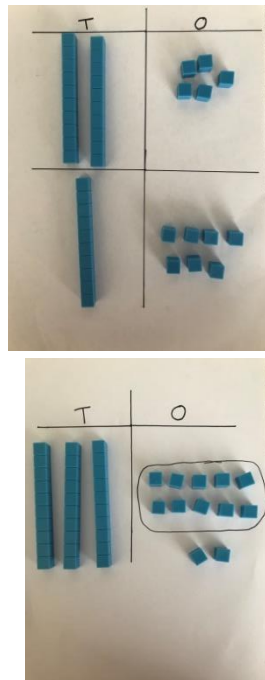
and 3 digit numbers



$$321 + 415 = 736$$

Column addition with regrouping

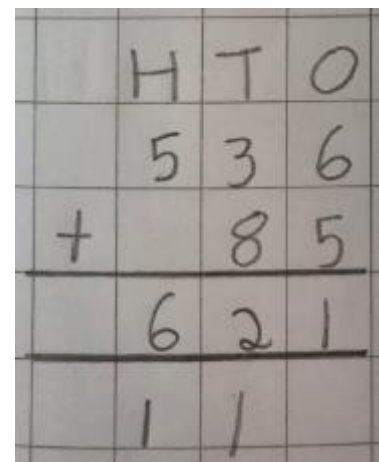
Model exchanging 10 ones

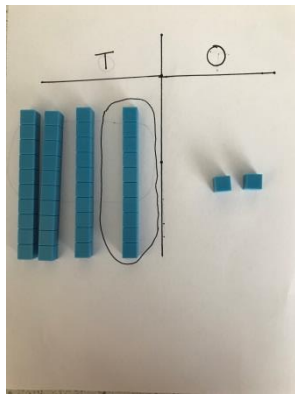
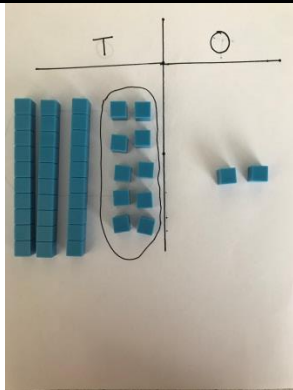


Children could draw a representation of the grid to further support their understanding, carrying the regrouped ten underneath the line

Then modelling where an exchange occurs between the tens and the hundreds

Start by partitioning the numbers before an informal method to show the exchange





Then modelling where an exchange occurs between the tens and the hundreds.

Dienes and place value counters are used

Note: progression with exchanging would be an exchange in one column, then two columns etc.

Year 4-6 Addition

Objective and strategy

Concrete

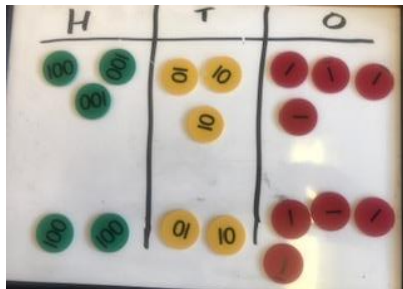
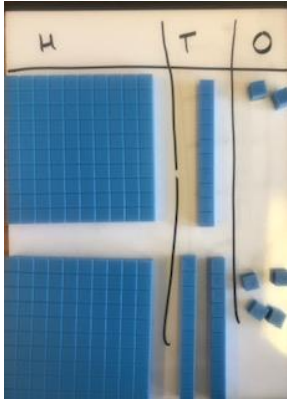
Pictorial

Abstract

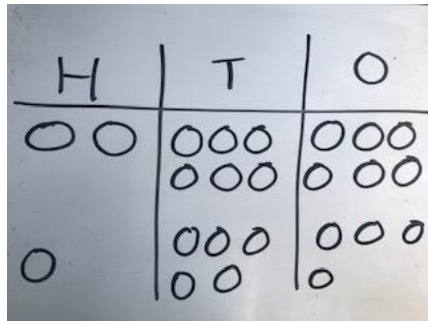
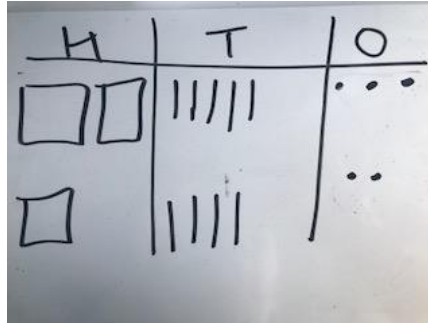
Vocabulary

Y4 - Add numbers with up to 4 digits

Children continue to use dienes or move onto place value counters to add. It is important that the similarities and differences between the two resources are discussed.



Draw representations using a place value grid of either Base 10 or place value counters.



Continue from previous work to **regroup** to hundreds as well as tens.

The headings only need to be there when this is initially being taught.

Relate to money and measures.

	T _h	H	T	O
	3	5	1	7
+		3	9	6
	3	8	1	3
		1	1	

Addition

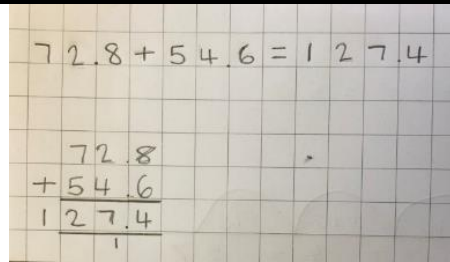
- Add
- More
- And
- Make
- Sum
- Total
- Altogether
- Double
- Near double
- Half
- Halve
- One more, two more...ten more, one hundred more
- How many more to make...?
- How many more is ... than ...?
- How much more is ...?
- Equals
- Is the same as
- Number bonds/pairs/facts
- Tens boundary/hundreds boundary/**ones**
- boundary/tenths boundary**
- Inverse

Y5 - Add numbers with more than 4 digits.

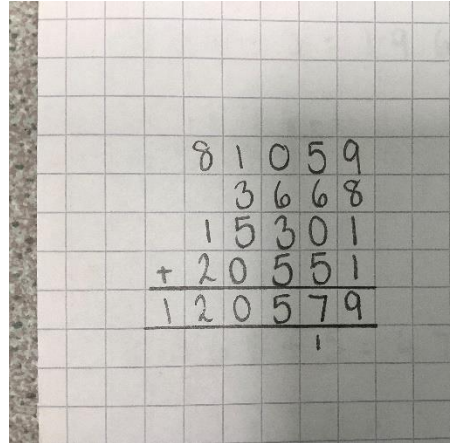
As Year 4 - introduce decimal place value counters and model exchange for addition.



Add decimals with 2 decimal places, including money

$$72.8 + 54.6 = 127.4$$


```
  72.8
+ 54.6
-----
 127.4
  1
```



```
  81059
   3668
 15301
+ 20551
-----
120579
  1
```

Y6 - Add several numbers of increasing complexity.

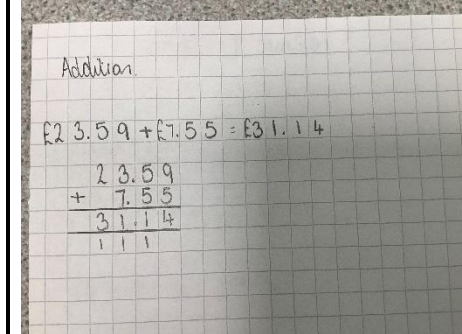
Include adding money, measures and decimals with different numbers of decimals

As Y5

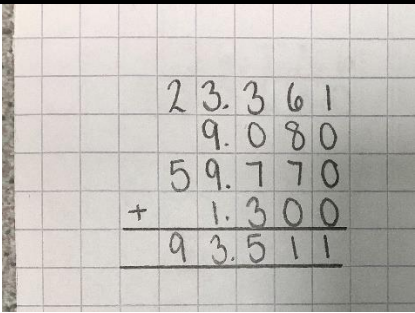
As Y5

Insert zeros for place holders

Addition

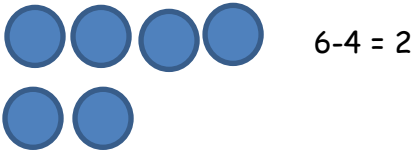
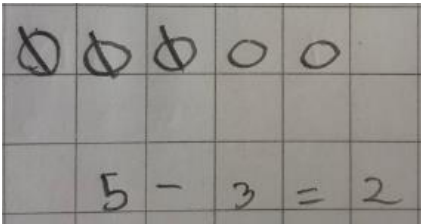
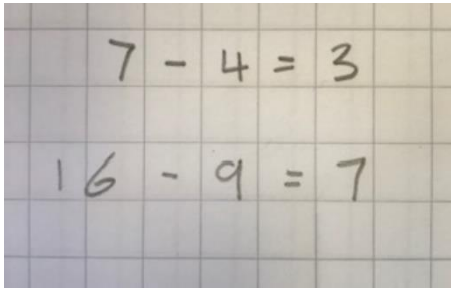
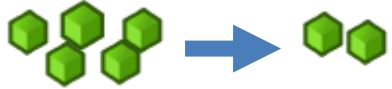


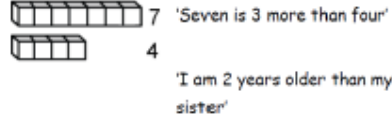
$$£23.59 + £7.55 = £31.14$$


```
  23.59
+  7.55
-----
 31.14
  1
```

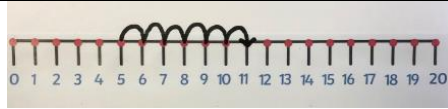
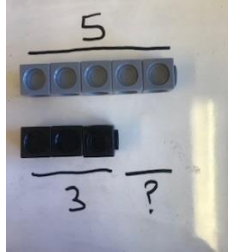
				
--	--	--	--	--

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \end{array}$$

Year 1 **Subtraction**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
Taking away ones	<p>Use physical objects (counters, cubes etc) to show how objects can be taken away.</p>  <p>$6 - 4 = 2$</p>	<p>Cross out objects to show what has been taken away.</p>  <p>$5 - 3 = 2$</p>	 <p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>	<p>Subtract Take away How many are left/left over? How many have gone? Ones less, two less, ten less... How many fewer is ... than ...? How much less is ...? Difference between</p>
Counting back	<p>Move objects away from the group counting backwards.</p> 	<p>Count back in ones using a number line.</p>  <p>$5 - 3 = 2$</p>	<p>Children can use a number line to help. Put 13 in your head. Count back 4. What number are you at?</p>	
Finding the difference	<p>Compare objects and amounts.</p>  <p>"5 is 2 more than 3"</p>	<p>Children represent objects pictorially.</p>  <p>Count on using a number line to find the difference.</p>	<p>The difference between 10 and 6 is 4. $10 - 4 = 6$</p> <p>Sophie has 12 football cards. Her sister has 5. How many more does Sophie have than her sister?</p>	

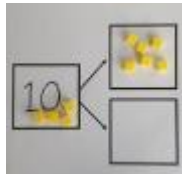
This can then form the basis of introducing bar modelling/part part whole



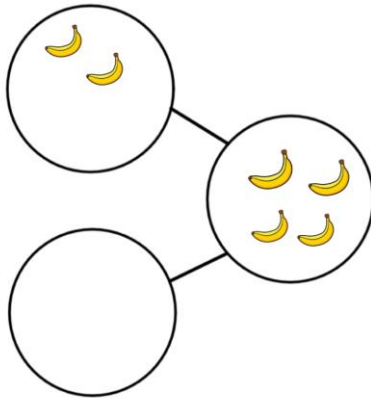
Children understand "Find the difference as subtraction"
 $11 - 6 = 5$

Part/whole model

Link to addition. Use the PPW model to model the inverse.

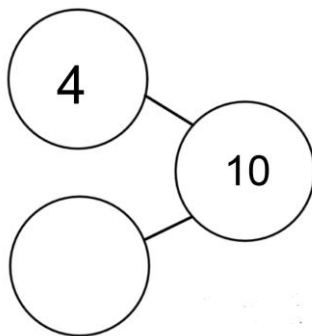


Use pictorial representation to show the part.



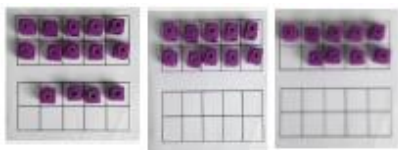
Move to using numbers within the part-part whole model.

$$10 - 4 = 6$$



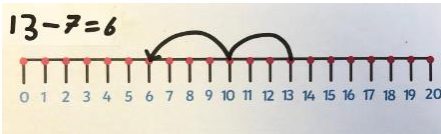
Make 10

14-9



Make 14 on the tens frame.
Take 4 away to make 10, then
take 5 away to make 9.


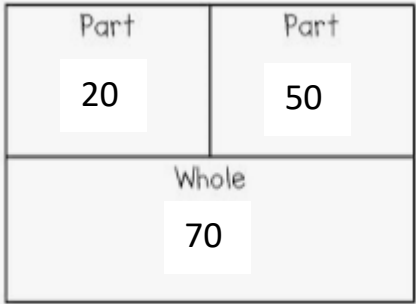
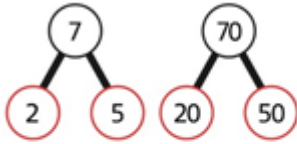
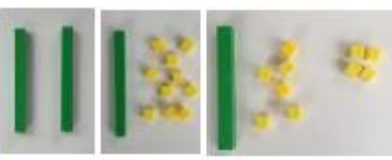
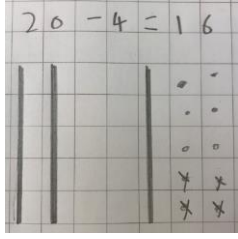
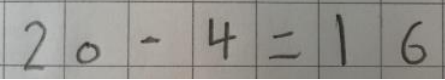
13-7

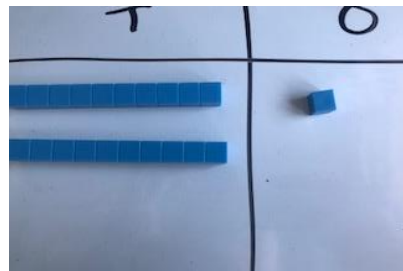
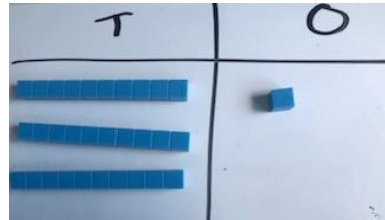
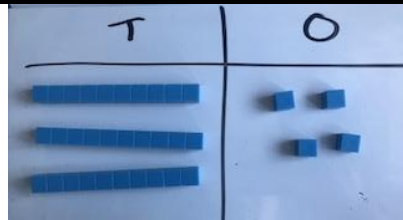


Jump back 3 to 10, then jump
back another 4.

16 - 8

How many do we need to take
away to make 10? How many do
we need to take away after?

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
Subtracting multiples of 10 Use known number bonds and unitising to subtract multiples of 10	Move objects away from the group  <i>8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.</i> $80 - 60 = 20$	  <i>7 tens subtract 5 tens is 2 tens.</i> $70 - 50 = 20$	$70 - 50 = 20$ $70 - 20 = 50$	Subtract Take away How many are left/left over? How many have gone? Ones less, two less, ten less, one hundred less... How many fewer is ... than ...? How much less is ...? Difference between Equals Is the same as Number bonds/pairs/facts Tens boundary
Regroup a ten into ten ones	Use a place value chart to show how to change a ten into ten ones $20 - 4$ 	$20 - 4$ 	$20 - 4 = 16$ 	
Partitioning to subtract without regrouping	Use dienes to show how to partition the number when subtracting without regrouping $34 - 13 = 21$	Children draw representations of dienes and cross off. $43 - 21 = 22$	$43 - 21 = 22$	



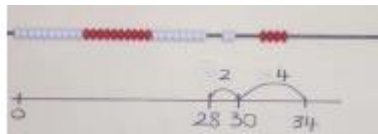
$$34 - 12 = 22$$

A handwritten number line on grid paper showing the subtraction 34 - 12 = 22. The number line starts at 0 and has markings at 10, 20, 30, and 40. A vertical line is drawn at 34. A bracket from 34 to 22 is labeled '12', and another bracket from 22 to 0 is labeled '22'.

Counting on

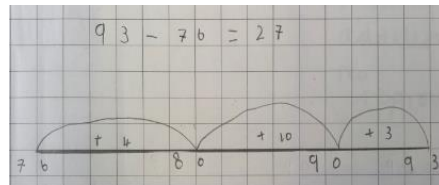
(Progression should be crossing one ten, crossing more than one ten, crossing the hundreds)

$$34 - 28$$



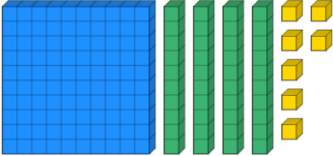
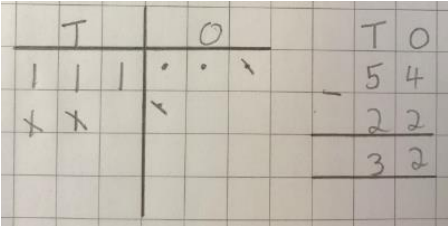
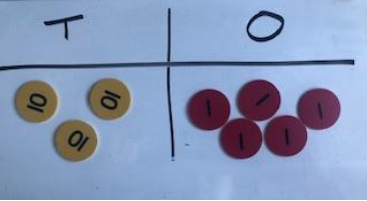
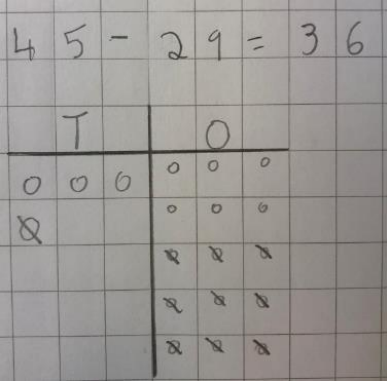
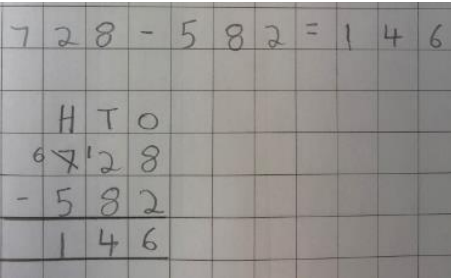
Use the bead string to model counting on to the next 10 from the smallest number and then the rest.

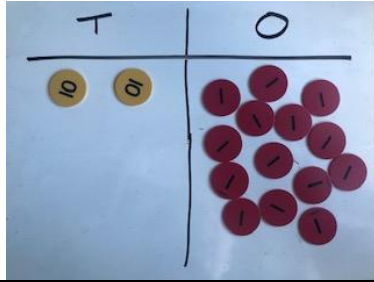
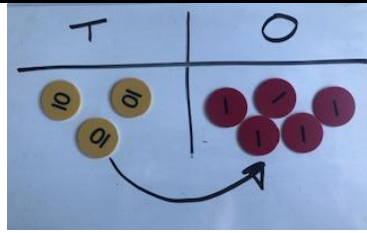
Use the number line to model counting on to the next 10 from the smallest number and then the rest.




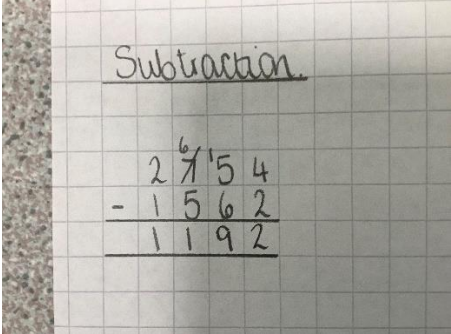
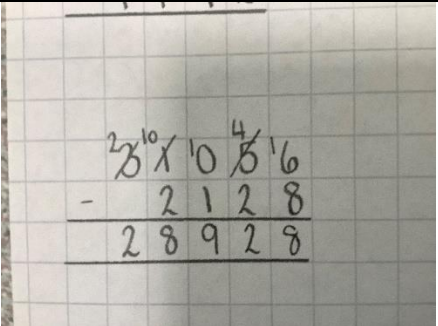
$$93 - 76 = 27$$

Year 3 **Subtraction**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Column subtraction without regrouping</p>	<p>Use Base 10 to model. 147 - 32</p> <p>Begin by showing 147 using Base 10</p>  <p>Then remove the 32</p>	<p>Draw representations to support understanding</p> 		<p>Subtract Take away How many are left/left over? How many have gone? Ones less, two less, ten less, one hundred less... How many fewer is ... than ...? How much less is ...? Difference between Equals Is the same as Number bonds/pairs/facts Tens boundary</p>
<p>Column subtraction with regrouping</p>	<p>Begin with Base 10 then move to place value counters modelling the exchange of a ten into ten ones.</p> <p>35 - 19</p> 	<p>Children may draw Base 10 or place counters and cross them off.</p> <p>45 - 29 = 36</p> 	<p>Then move to formal method.</p> 	<p>Hundreds boundary</p>



Year 4-6 **Subtraction**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Subtracting tens and ones.</p> <p>Year 4 subtract with up to 4 digits</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p>234 - 179</p> <p>Model process of exchange using Base 10 or Place Value counters and then move to place value counters.</p>	<p>Children to draw the place value counters and show the exchange.</p> 		<p>Subtract</p> <p>Take away</p> <p>How many are left/left over?</p> <p>How many have gone?</p> <p>Ones less, two less, ten less, one hundred less...</p> <p>How many fewer is ... than ...?</p> <p>How much less is ...?</p> <p>Difference between</p> <p>Equals</p> <p>Is the same as</p> <p>Number bonds/pairs/facts</p> <p>Missing number</p>
<p>Year 5 subtract with at least 4 digits, including measures and money.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals.</i></p> <p><i>Include where</i></p>	<p>As Year 4</p>	<p>As Year 4</p>		<p>Tens boundary, hundreds boundary, ones boundary, tenths boundary</p> <p>Inverse</p>

the decimal needs to be aligned.


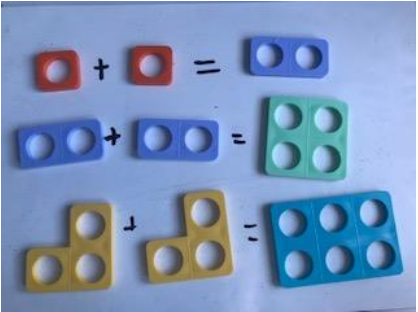
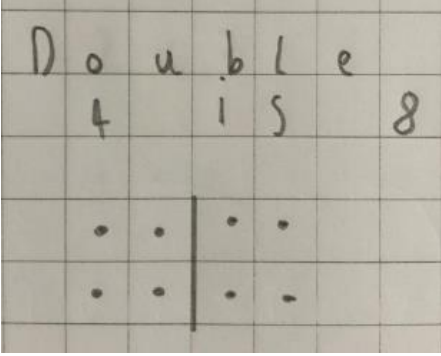
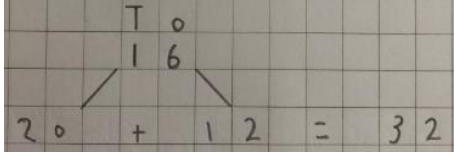
$$\begin{array}{r} \overset{6}{7} \overset{10}{\cancel{1}} \overset{6}{6} \overset{8}{\cancel{9}} . 00 \\ - \quad 372.50 \\ \hline 6796.50 \end{array}$$


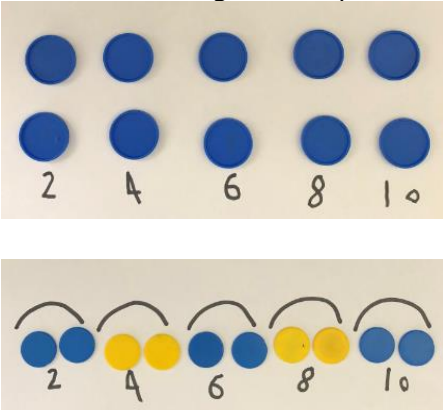
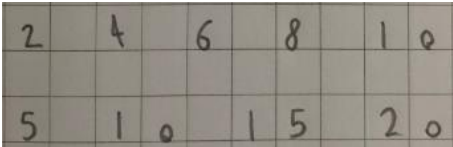

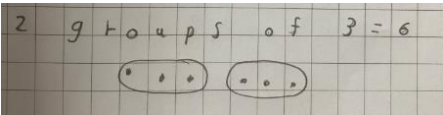
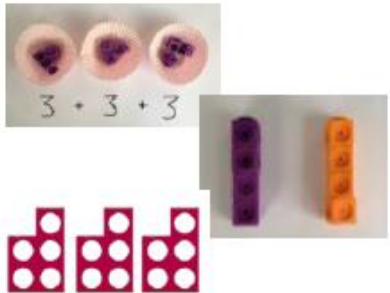
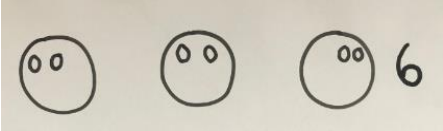
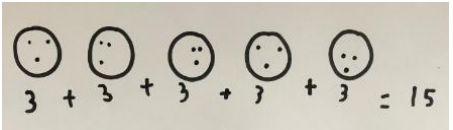
Year 6
Subtract with increasingly large and more complex numbers and decimal values


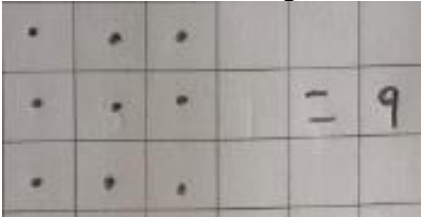
$$\begin{array}{r} \overset{0}{\cancel{1}} \overset{14}{\cancel{5}} \overset{9}{\cancel{0}} \overset{1}{6} 99 \\ - \quad 89949 \\ \hline 60750 \end{array}$$

$$\begin{array}{r} \overset{9}{\cancel{1}} \overset{3}{\cancel{0}} \overset{5}{5} \overset{3}{\cancel{4}} 19 \\ - \quad 36.080 \\ \hline 69.339 \end{array}$$

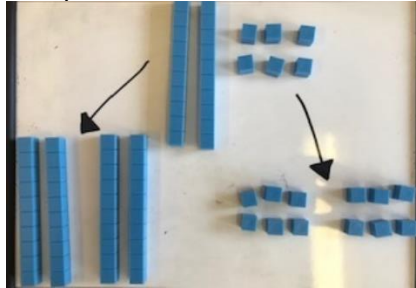
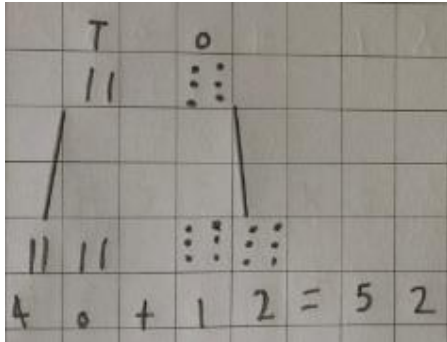
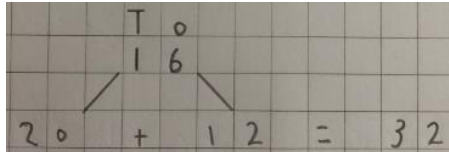
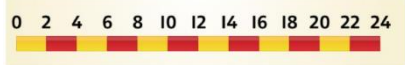
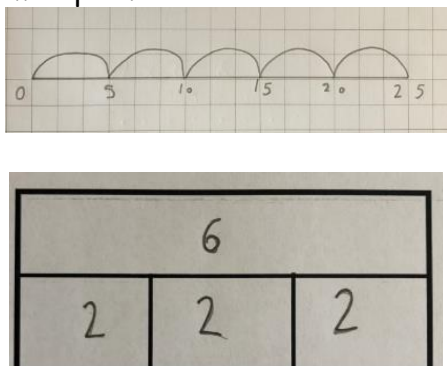
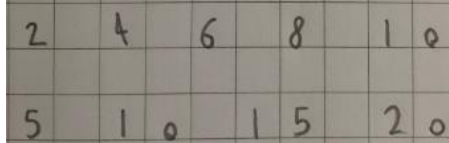
Year 1 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.</p>  <p>Double 4 is 8</p> 	<p>Draw pictures to show how to double numbers.</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 	<p> multiplication multiply multiplied by multiple doubling array number patterns </p>

<p>Counting in multiples</p>	<p>Count the groups as children are skip counting. Children may use their fingers as they are skip counting.</p> 	<p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud. Write sequences with multiples of numbers.</p> 	
<p>Making equal groups and counting the totals</p>	<p>Use manipulatives to create equal groups.</p>  <p><input type="text"/> x <input type="text"/> = 8</p>	<p>Draw and make representations. 2 groups of 3 = 6</p> 	<p>$2 \times 4 = 8$</p>	
<p>Repeated addition</p>	<p>Use different objects to add equal groups.</p> 	<p>Use pictorial representations, including number lines, to solve problems. e.g. There are 2 sweets in each bag. How many sweets are in 3 bags?</p> 	<p>Write addition sentences to describe objects and pictures.</p> 	

Understanding arrays	Use objects laid out in arrays to find answers. 	Draw representations of arrays to show understanding. 	$2 \times 3 = 6$ $2 \times 5 = 10$	




Year 2 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Doubling</p>	<p>Model doubling using dienes and place value counters.</p>  <p>Double 26 =</p> $40 + 12 = 52$	<p>Draw pictures to represent how to double numbers.</p> 	<p>Partition a number then double each part before recombining it.</p> 	<p>multiplication multiply multiplied by multiple groups of times once, twice, three times ... ten times repeated addition equal groups of doubling halving array row, column number patterns multiplication table multiplication fact, division fact</p>
<p>Counting in multiples of 2, 5 and 10 from zero (repeated addition)</p>	<p>Count the groups as children are skip counting. Use counting sticks in class</p> 	<p>Number lines and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud. Write sequences with multiples of numbers.</p> 	



5	x	4	=	20
5	x	3	=	15

Multiplication is commutative

Create arrays using counters, cubes or Numicon.

Pupils should understand that arrays can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.

Use pictorial representations of arrays to show different calculations and explore commutativity.

2	x	3	=	3	x	2
.
.
.

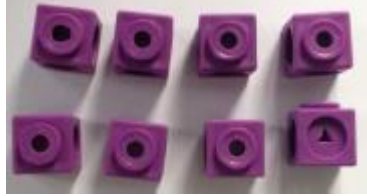
.
.
2	x	5	=	10
5	x	2	=	10
10	=	2	x	5
10	=	5	x	2

Use an array to write multiplication and reinforce repeated addition.

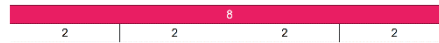
.	.					
.	.					
.	.					
2	+	2	+	2	=	6
3	+	3	=	6		
3	x	2	=	6		
2	x	3	=	6		

Using the inverse

This should be taught alongside division, so pupils learn how they work alongside each other.



Represent in the form of a bar model.



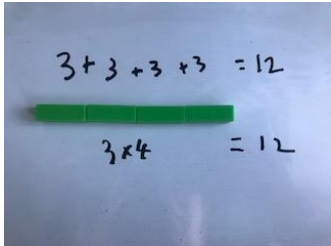
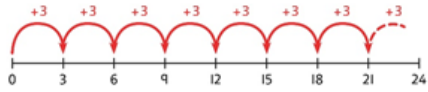
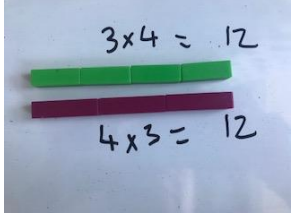

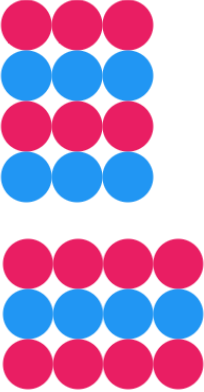
$2 \times 4 = 8$	$8 = 2 \times 4$
$4 \times 2 = 8$	$8 = 4 \times 2$
$8 \div 2 = 4$	$4 = 8 \div 2$
$8 \div 4 = 2$	$2 = 8 \div 4$

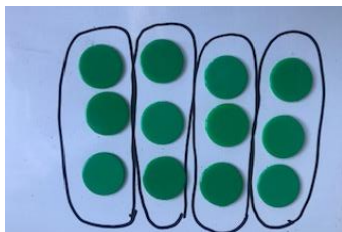
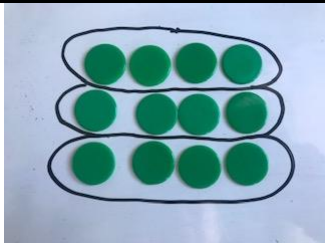
Ensure number sentences with = at the start and end are taught.

Show all 8 related fact family sentences.

$2 \times 4 = 8$	$8 = 2 \times 4$
$4 \times 2 = 8$	$8 = 4 \times 2$
$8 \div 2 = 4$	$4 = 8 \div 2$
$8 \div 4 = 2$	$2 = 8 \div 4$

Year 3 **Multiplication**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Continue to understand equal grouping and repeated addition</p>	<p>Children build on previous learning about equal groups and the relationship with repeated addition. Children will recognise examples and non-examples</p> 	<p>Children understand the link between repeated addition and multiplication</p>  <p>$3+3+3+3+3+3+3+3 = 24$ $3 \times 8 = 24$</p>	<p>Once children have experienced concrete and pictorial opportunities to support their conceptual understanding, they can use recall of multiplication facts</p> <p>$3 \times 8 = 24$</p>	<p>multiplication multiply multiplied by multiple, factor groups of times product once, twice, three times ... ten times repeated addition equal groups of doubling halving array row, column number patterns multiplication table multiplication fact, division fact</p>
<p>Using commutativity to support understanding of the times tables</p>	  <p>Children recognise that arrays demonstrate commutativity</p>	<p>Children could draw arrays.</p> 	<p><i>I need to work out 5 groups of 8.</i></p> <p><i>I know that $5 \times 8 = 40$ Therefore, I know that 8×5 also = 40.</i></p> <p>$2 \times 4 = 8$</p> <p>$20 \times 4 = 80$</p>	

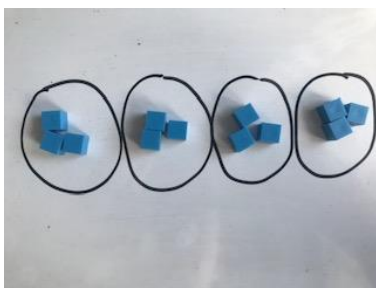


There are 3 groups of 4
There are 4 groups of 3
There are 12 counters altogether

Use known facts to multiply 10s

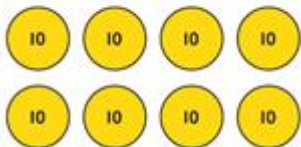
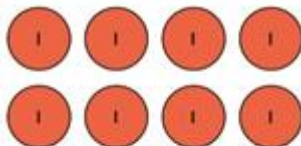
Use Base 10 to explore the relationship between known times-tables and multiples of 10

Make 4 groups of 3 ones



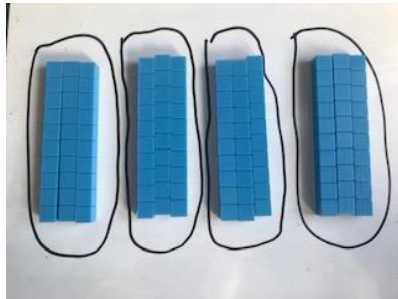
Make 4 groups of 3 10s

Children understand how unitising 10s supports multiplying by multiples of 10



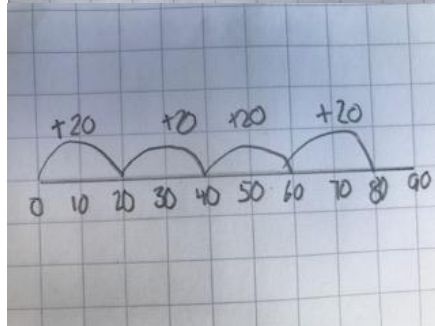
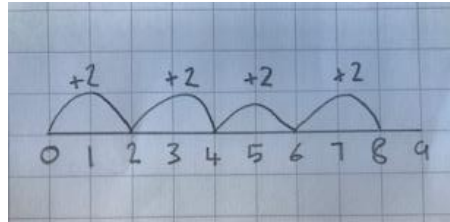
4 groups of 2 ones is 8 ones
4 groups of 2 tens is 8 tens

$$2 \times 4 = 8$$



What's the same? What's different?

20×4 is 10 times larger so the answer is $8 \times 10 = 80$

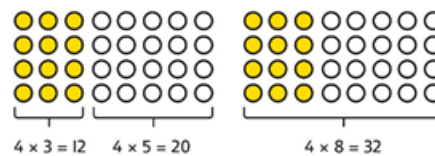


Understanding and using partitioning in multiplication

Represent numbers using dienes or place value counters.

Tens	Ones
10 10 10 10	1 1
10 10 10 10	1 1
10 10 10 10	1 1

42×3 is 3 groups of 10 and 3 groups of 2.



$$4 \times 3 = 12$$

$$4 \times 5 = 20$$

$$12 + 20 = 32$$

$$4 \times 8 = 32$$





Use partitioning to multiply 2-digit numbers by a single digit

$$18 \times 6$$

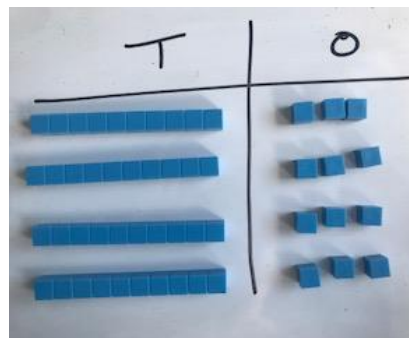
$$10 \times 6 = 60$$

$$8 \times 6 = 48$$

$$18 \times 6 = 60 + 48 = 108$$

Tens	Ones
	
	

27×2 is 2 groups of 10 and 2 groups of 7.



$18 \times 6 = 2$

$18 \times 6 = 10 \times 6 + 8 \times 6$
 $= 60 + 48$
 $= 108$

$10 \times 6 = 60$
 $8 \times 6 = 48$

$18 \times 6 = 10 \times 6 + 8 \times 6$
 $= 60 + 48$
 $= 108$

	T	O
	3	4
x		2
	6	8

Year 4 Multiplication

Objective and strategy

Concrete

Pictorial

Abstract

Vocabulary

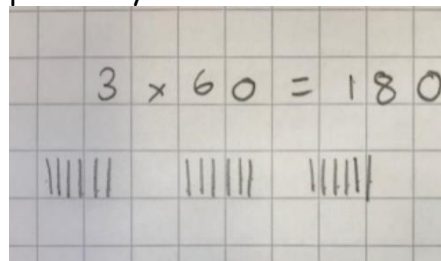
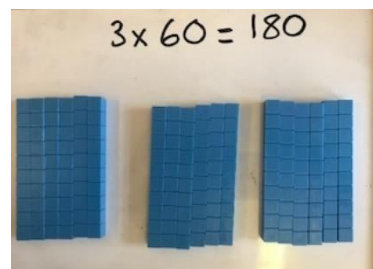
Multiplying by multiples of 10 and 100

Use Base 10 to understand how to multiply by multiples of 1, 10 and 100

Represent concrete resources pictorially

Children use known facts and understanding of place value and commutativity to multiply mentally

multiplication
 multiply
 multiplied by
 multiple, factor
 groups of
 times
 product





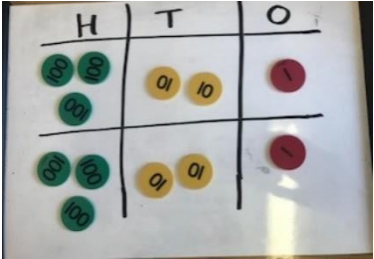
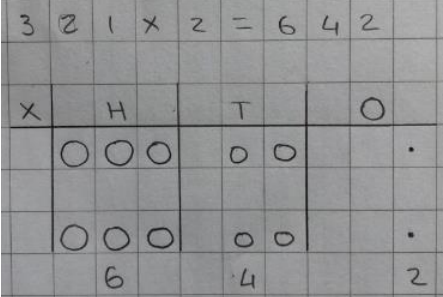
$4 \times 7 = 28$

$4 \times 70 = 280$

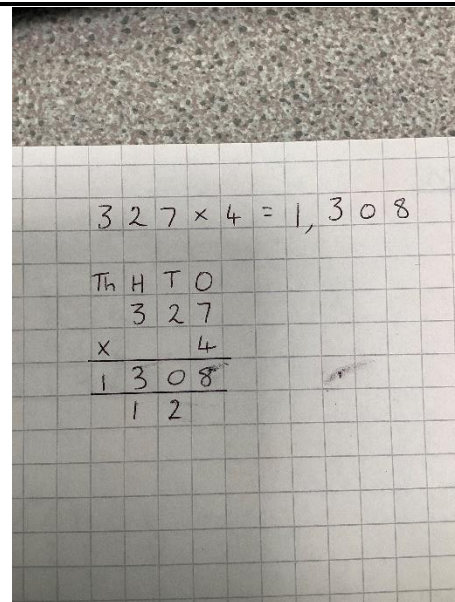
$40 \times 7 = 280$

$4 \times 700 = 2800$

once, twice, three times ... ten times
 repeated addition
 doubling

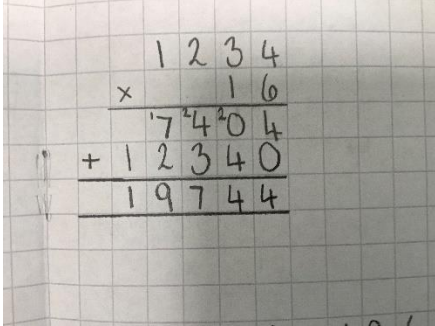
	<p>This could lead to a discussion that 3×6 is ten times smaller etc.</p>		$400 \times 7 = 2800$	<p>array row, column number patterns multiplication table multiplication fact, division fact</p>
<p>Use knowledge of commutativity and factors to simplify some multiplications</p>	 <p>Each sheet has 2×5 stickers. There are 3 sheets.</p> <p>There are $5 \times 2 \times 3$ stickers in total.</p> $5 \times 2 \times 3 = 30$ $\underbrace{\quad\quad}_5 \times 3 = 30$ $10 \times 3 = 30$	 <p>$2 \times 6 \times 10 = 120$ $12 \times 10 = 120$</p> <p>$10 \times 6 \times 2 = 120$ $60 \times 2 = 120$</p>	$24 \times 5 = 12 \times 2 \times 5$ $12 \times 2 \times 5 = 12 \times 10 = 120$ Therefore $24 \times 5 = 120$	
<p>Grid method recap 2 digit by 1 digit</p> <p>Then 3 digits by 1 digit</p>	<p>Use Place Value counters as in Y3</p>	<p>As Y3</p>		
<p>Column multiplication</p>	<p>Children can continue to be supported by Place Value counters at the multiplication stage. It is important at this stage that they multiply the ones first.</p> <p>321×2</p> 			

It is important to model the corresponding written multiplication next to it.

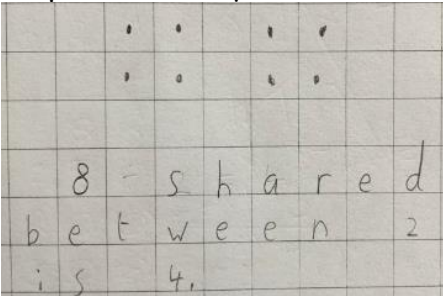


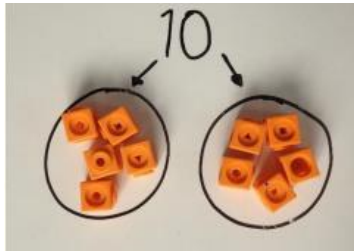
Year 5/6 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
Column multiplication for 3 and 4 digit by 1 digit	As Y4 extending up to 4 digit numbers	As Y4 extending up to 4 digit numbers	As Y4 extending up to 4 digit numbers	

Column multiplication	Manipulatives may still be used with the corresponding calculation alongside.			multiplication multiply multiplied by multiple, factor groups of times product once, twice, three times ... ten times repeated addition doubling array row, column number patterns multiplication table multiplication fact, division fact

Year 1 **Division**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
Division as Sharing	I have 10 cubes can you share them equally between two groups?	Children draw pictures or shapes to share quantities 	"12 shared between 3 is 4." There is no requirement to use the symbol for division in Y1. This could be verbalised or written using stem sentences.	division dividing grouping sharing



Division as grouping

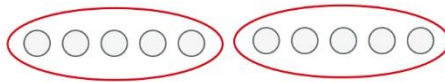
It is vital that children are given the opportunity to look at division as grouping rather than just sharing.

Divide quantities into equal groups

$$10 \div 2 = 5$$



Represent a whole and work out how many equal groups.

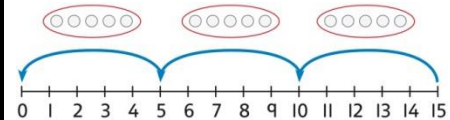


*There are 10 in total.
There are 5 in each group.
There are 2 groups.*

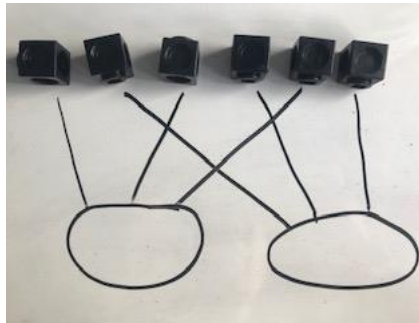

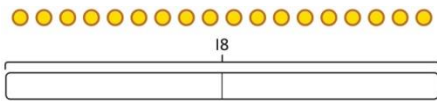
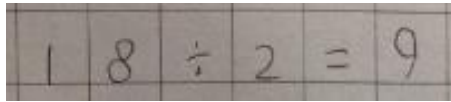
There is no requirement to use the symbol for division in Y1.

This could be verbalised or written using stem sentences.

Children may relate grouping to counting back in steps of 2, 5 or 10.



Year 2 **Division**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Division as sharing</p>	<p>Start with a whole and share into equal parts, one at a time.</p>  <p>12 shared equally between 2. They get 6 each.</p> <p>Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared</p>	<p>Represent the objects shared into equal parts using a bar model.</p>  <p>20 shared into 5 equal parts. There are 4 in each part.</p> <p>Use a bar model to support understanding of the division.</p>  <p>$18 \div 2 = 9$</p>	<p>Children will need to be introduced to the symbol for division</p>  <p>Children should be able to verbalise this as 18 divided into 2 groups means there are 9 in each group.</p>	<p>division dividing, divide, divided by, divided into grouping sharing, share, share equally left, left over one each, two each, three each ... ten each group in pairs, threes ... tens equal groups of</p> <p>multiplication table multiplication fact, division fact</p>



6 shared equally between 3.
They get 2 each.

Division as grouping

Children understand how to make equal groups from a whole.



8 divided into 4 equal groups.
There are 2 in each group.

Children understand the relationship between grouping and the division statements.

$$12 \div 3 = 4$$



$$12 \div 4 = 3$$



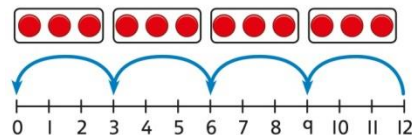
$$12 \div 6 = 2$$



$$12 \div 2 = 6$$



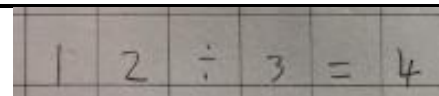
Children understand how to relate division by grouping to repeated subtraction.




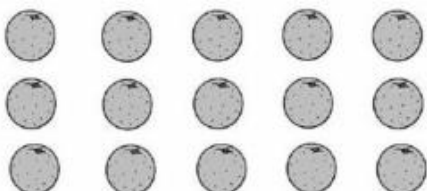
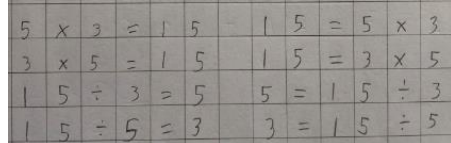
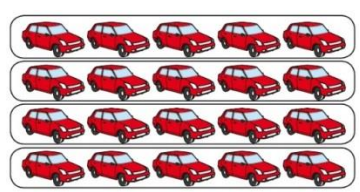
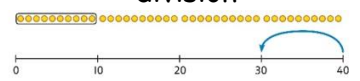
There are 4 groups now.

12 divided into groups of 3.

$$12 \div 3 = 4$$

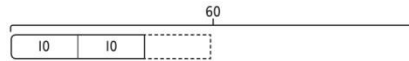


Children should be able to verbalise this as 12 grouped into 3s = 4 groups

<p>Division with arrays</p>	<p>Link multiplication to division by creating an array and thinking about the number sentences that can be created.</p>  $15 \div 3 = 5$ $15 \div 5 = 3$ $3 = 15 \div 5$ $5 = 15 \div 3$ $3 \times 5 = 15$ $5 \times 3 = 15$ $15 = 3 \times 5$ $15 = 5 \times 3$	<p>There are 4 groups.</p> <p>Draw an array and use lines to split it into groups to make multiplication and division sentences.</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> 	
<p>Use known times-tables to solve division</p>	<p>Understand the relationship between multiplication facts and division.</p> 	<p>Link equal grouping with repeated subtraction and known times-table facts to support division</p>  <p>40 divided by 4 is 10.</p> <p>Use a bar model to support understanding of the link</p>	<p>Relate times-table knowledge directly to division.</p> $1 \times 10 = 10$ $2 \times 10 = 20$ $3 \times 10 = 30$ $4 \times 10 = 40$ $5 \times 10 = 50$ $6 \times 10 = 60$ $7 \times 10 = 70$ $8 \times 10 = 80$ <div style="border: 1px solid orange; border-radius: 15px; padding: 10px; display: inline-block; margin-top: 10px;"> <p>I used the 10 times-table to help me. 3 x 10 = 30.</p> </div>	

4 groups of 5 cars is 20 cars
in total.
20 divided by 4 is 5.


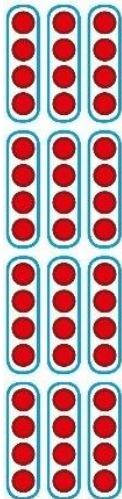
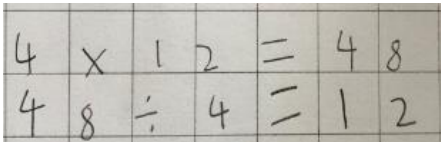
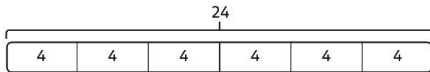
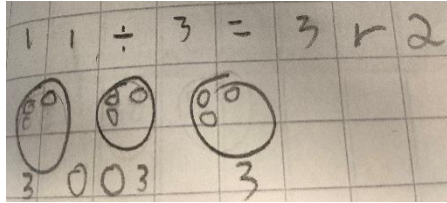
between times-table knowledge
and division.



I know that 3 groups of 10
makes 30, so I know that 30
divided by 10 is 3.

$$3 \times 10 = 30 \quad \text{so} \quad 30 \div 10 = 3$$

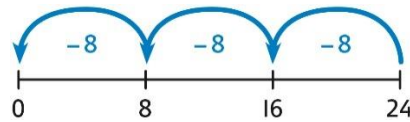
Year 3 **Division**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Use known times-tables to solve division</p>	<p>Use knowledge of known times-tables to calculate divisions.</p>  <p><i>24 divided into groups of 8. There are 3 groups of 8.</i></p>	<p>Use knowledge of known times-tables to calculate divisions.</p>  <p>$48 \div 4 = 12$</p> <p><i>48 divided into groups of 4. There are 12 groups.</i></p>  <p>A bar model may represent the relationship between sharing and grouping.</p> 	<p>Use knowledge of known times-tables to calculate divisions.</p> <p>Solving examples with remainders</p> 	<p>division dividing, divide, divided by, divided into left, left over, remainder grouping sharing, share, share equally one each, two each, three each ... ten each group in pairs, threes ... tens equal groups of</p> <p>multiplication table multiplication fact, division fact</p>

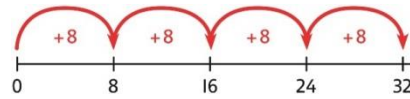
$$24 \div 4 = 6$$

$$24 \div 6 = 4$$

Children understand how division is related to both repeated subtraction and repeated addition.



$$24 \div 8 = 3$$

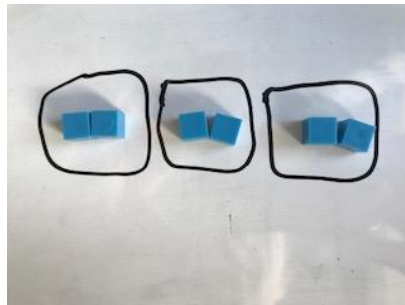


$$32 \div 8 = 4$$

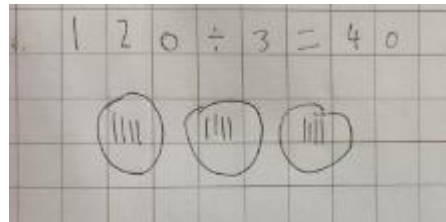
Use known facts to divide multiples of 10

Use place value equipment to understand how to divide by unitising.

Divide 6 ones between 3 groups



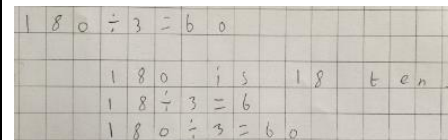
Divide multiples of 10 by unitising.



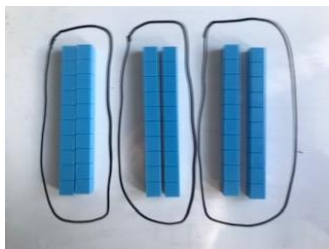
12 tens shared into 3 equal groups.

4 tens in each group.

Divide multiples of 10 by a single digit using known times-tables.



Now divide 6 tens between 3 groups



What is the same? What is different?

Divide a 2 digit number by a 1 digit number using partitioning

Children explore dividing 2-digit numbers by using place value equipment.

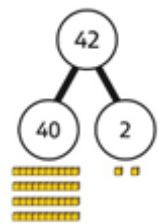


$42 \div 2 = ?$

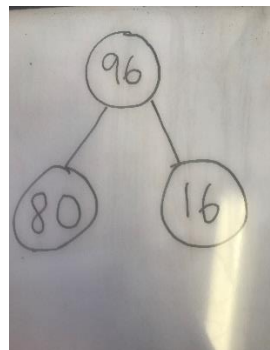


Children explore which partitions support particular divisions.

$42 \div 2 =$



$96 \div 8$



$69 \div 3 =$

69 is partitioned into 60 and 9

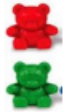
6	9	÷	3	=	2	3
6	0	÷	3	=	2	0
9	÷	3	=	3		
2	0	+	3	=	2	3

Children have an awareness of remainders

Using the word remainder not the letter r

$14 \div 3 =$

Divide objects between groups and see how much is left over



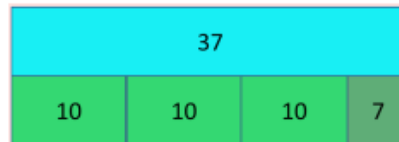
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.


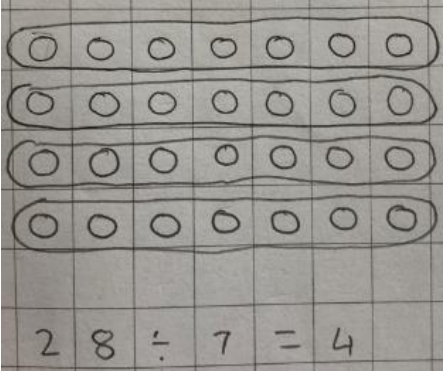
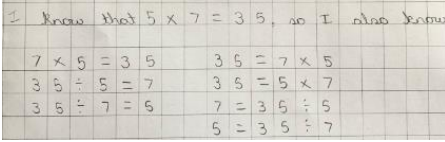
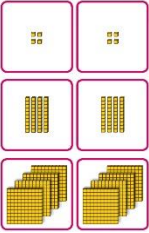
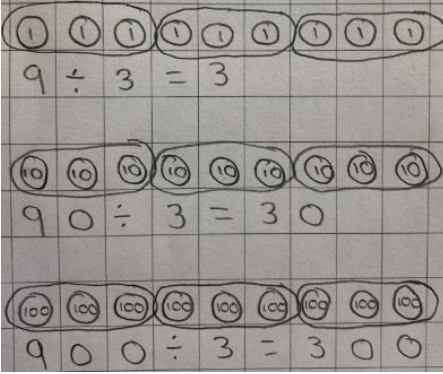
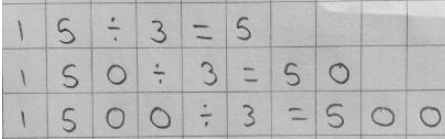


Use bar models to show division with remainders.



$29 \div 8 = 3$ with 1 left over
 $29 \div 8 = 3$ remainder 1

Year 4 **Division**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
<p>Understand the relationship between multiplication and division, including times-tables</p>	<p>Use objects to explore families of multiplication and division facts.</p>  <p>$4 \times 6 = 24$ 24 is 6 groups of 4. 24 is 4 groups of 6.</p> <p>24 divided by 6 is 4. 24 divided by 4 is 6.</p>	<p>Represent divisions using an array.</p>  <p>$24 \div 6 = 4$</p>	<p>Understand families of related multiplication and division facts.</p> 	<p>division dividing, divide, divided by, divided into left, left over, remainder grouping sharing, share, share equally one each, two each, three each ... ten each group in pairs, threes ... tens equal groups of</p> <p>multiplication table multiplication fact, division fact</p>
<p>Dividing multiples of 10 and 100 by a single digit</p>	<p>Use place value equipment to understand how to use unitising to divide.</p>  <p>8 ones divided into 2 equal groups 4 ones in each group</p>	<p>Draw the place value counters to support with calculation.</p>  <p>$9 \div 3 = 3$ $90 \div 3 = 30$ $900 \div 3 = 300$</p> <p>$9 \div 3 = 3$</p>	<p>Use known facts to divide 10s and 100s by a single digit.</p> 	

8 tens divided into 2 equal groups
4 tens in each group

8 hundreds divided into 2 equal groups
4 hundreds in each group

9 tens divided by 3 is 3 tens.
9 hundreds divided by 3 is 3 hundreds.

Informal methods of dividing 2-digit and 3-digit numbers by a single digit

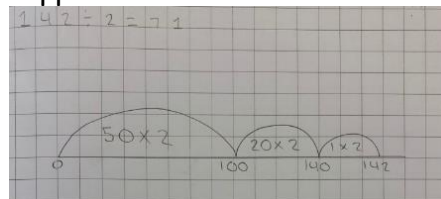
Use place value equipment to explore why different partitions are needed.

$$42 \div 3 = ?$$

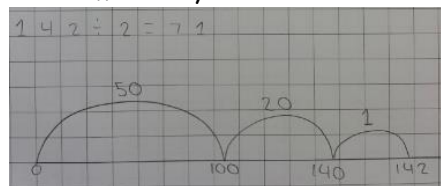
I will split it into 30 and 12, so that I can divide by 3 more easily.



Children use a number line to support



Then move to,

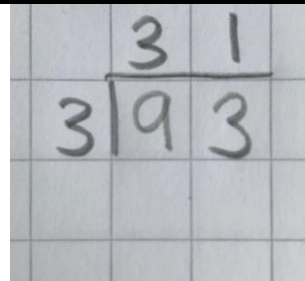
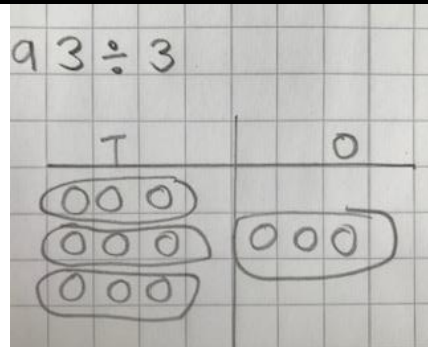
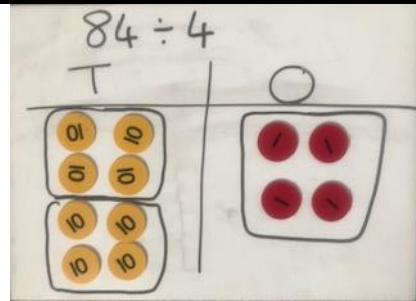


Represent how to partition flexibly where needed.

8	4	÷	7	=	1	2
7	0	÷	7	=	1	0
1	4	÷	7	=	2	
1	0	+	2	=	1	2

84 is partitioned into 70 and 14, as these are both divisible by 7.

Short Division



Year 5 **division**

Objective and strategy

Concrete

Pictorial

Abstract

Vocabulary

Short Division

452 ÷ 4 =

H	T	O
100 100 100 100	10 10 10 10 10	1 1

452 ÷ 4 =

H	T	O
100 100 100 100	10 10 10 10 10	1 1

452 ÷ 4 = 113

H	T	O
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1 1

1 1 3

Children can draw the place value counters.

452 ÷ 4

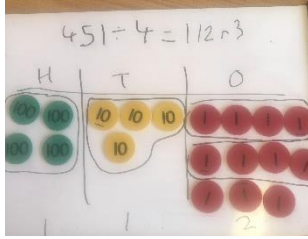
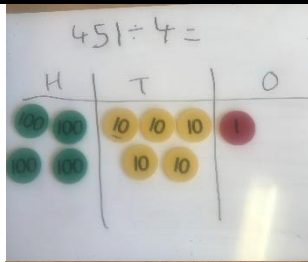
H	T	O
00 00	00 00 00	0000 0000

256 ÷ 4 = 64

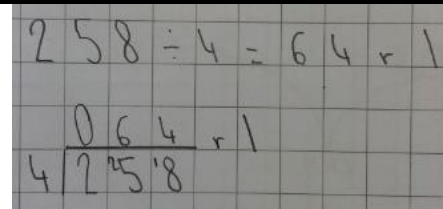
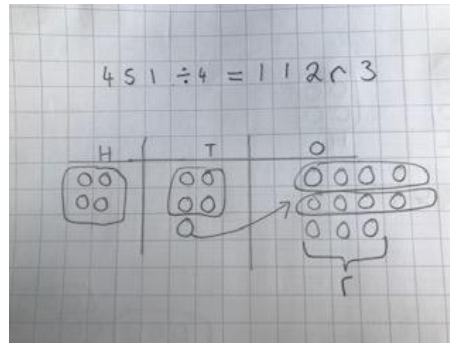
0	6	4
4	2	56

division
dividing, divide, divided by, divided into
left, left over, remainder
grouping
sharing, share, share equally
one each, two each, three each ... ten each
group in pairs, threes ... tens
equal groups of
multiplication table
multiplication fact, division fact

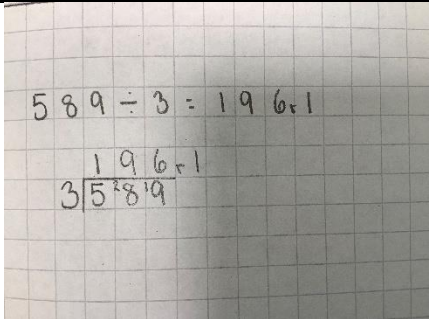
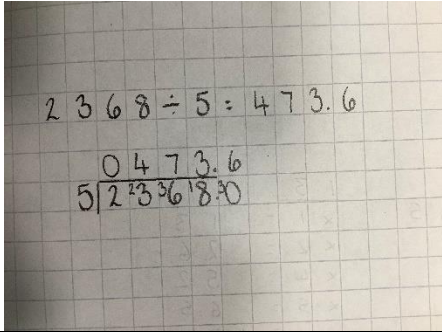
Short Division method with remainders represented using r



Children can draw the place value counters.



Year 6 **division**

Objective and strategy	Concrete	Pictorial	Abstract	Vocabulary
Short Division method with remainders	Manipulatives may still be used with the corresponding calculation alongside.		 	division dividing, divide, divided by, divided into left, left over, remainder grouping sharing, share, share equally one each, two each, three each ... ten each group in pairs, threes ... tens equal groups of multiplication table multiplication fact, division fact
Short Division (Bus stop) method with decimal remainders	Manipulatives may still be used with the corresponding calculation alongside.			

Long division

$$2145 \div 13 = 165$$

$$\begin{array}{r} 165 \\ 13 \overline{) 2145} \\ \underline{- 1300} \quad (x100) \\ 7845 \\ \underline{- 650} \quad (x50) \\ 195 \\ \underline{- 130} \quad (x10) \\ 65 \\ \underline{- 65} \quad (x5) \\ 0 \end{array}$$

$$\begin{array}{r} 13 \\ \times 1 = 13 \\ \times 2 = 26 \\ \times 4 = 52 \\ \times 5 = 65 \\ \times 10 = 130 \end{array}$$